Assignment2

March 6, 2026

[2]:
```
# remember to include these two lines of code at the start of your document!
# they allow you to receive multiple outputs from a single code chunk.
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

0.1 Assignment 2

Please submit your .ipynb file and .pdf file.

0.2 1. (11 points) Direction Fields of Differential Equations

Plot the direction field for the differential equation:

$$y' = x^2y - \frac{1}{2}x^2$$

Then add on the solution curves for the following initial conditions:

- $y(0) = 1.8$
- $y(-1) = -1$
- $y(1) = 0$
- $y(1) = 0.3$

0.2.1 1.1 (3 points) Plot the direction field of the differential equation

[3]:
```
#
# Solution
#
```

0.2.2 1.2 (4 points) Solving the differential equation using initial conditions given.

- 4 intial conditions are given, you will have 4 solutions.

[4]:
```
#
# Solution
#
```

0.2.3 1.3 (4 points) Creating plots of the solution curves and add them to the direction field

- For the 4 solutions, 4 solution curves will be added to the direction field.

[5]:
```
#
# Solution
#
```

0.3 2 (9 points) Sequences

Plot the first 30 terms of the flowing sequences:

2.1 (3 points)

$$a_n = \frac{n!}{n^{(n/3)}}$$

[6]:
```
#
# Solution
#
```

0.3.1 2.2 (3 points)

$$a_n = \frac{\sin(n\pi/2)}{n}$$

[7]:
```
#
# Solution
#
```

0.3.2 2.3 (3 points) Monotone bounded sequence:

$$a_n = 1 - 1/n$$

[8]:
```
#
# Solution
#
```

0.4 3. (10 points) Integral Test and P-series

A **p-series** is a series of the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p}.$$

The theorem: - The series **converges** if $p > 1$. - The series **diverges** if $p \leq 1$.

0.4.1 3.1 (3 points) Plot the partial sums of p-series from 1 to 100 for different values of $p: 1/3, 1, 3$.

[9]:
```
#
# Solution
#
```

We consider the improper integral

$$\int_1^\infty x^{-p}\,dx.$$

This integral converges if and only if $p > 1$. For $p \neq 1$, we know

$$\int_1^\infty x^{-p}dx = \lim_{b\to\infty} \frac{b^{1-p}-1}{1-p}.$$

- If $p > 1$, then $1-p < 0$ and $b^{1-p} \to 0$, so the integral converges. - If $p \leq 1$, the integral diverges.

0.4.2 3.2 (3 points) Plotting the Improper Integral as a Function of B

$$F_p(B) = \int_1^B x^{-p}dx$$

for several values of $p:\ p = 1/3,\ p = 1$, and $p = 3$.

[10]:
```
#
# Solution
#
```

The **Integral Test** says:

Let $f(x)$ be a positive, continuous, decreasing function on $[1, \infty)$, and suppose $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ and the improper integral $\int_1^\infty f(x)dx$ either both converge or both diverge.

We now apply this to p-series and some related examples. ### Example: $\sum \frac{1}{n^{3/2}}$

Here $a_n = 1/n^{3/2}$ and $f(x) = x^{-3/2}$. - f is positive, continuous, and decreasing on $[1, \infty)$. - We know $\int_1^\infty x^{-3/2}dx$ converges (since $p = 3/2 > 1$). - By the Integral Test, $\sum 1/n^{3/2}$ converges.

0.4.3 3.3 (2 points) Plot the partial sums of the series $a_n = 1/n^{3/2}$ from 1 to 50

[11]:
```
#
# Solution
#
```

0.4.4 Example $a_n = 1/n^{4/5}$

Here $a_n = 1/n^{4/5}$ and $f(x) = x^{-4/5}$. - f is positive, continuous, and decreasing on $[1, \infty)$. - We know $\int_1^\infty x^{-4/5}dx$ diverges (since $p = 4/5 \leq 1$). - By the Integral Test, $\sum 1/n^{4/5}$ diverges.

0.4.5 3.4 (2 points) Plot the partial sums of the series $a_n = 1/n^{4/5}$ from 1 to 50

[12]:
```
#
# Solution
#
```

0.5 (Bonut Points 10) Multivariable Functions and Contour Map

A single-variable function looks like:

$$y = f(x)$$

It takes one input and gives one output. You can draw it as a curve in the plane. A two-variable function looks like:

$$z = f(x, y)$$

It takes two inputs and gives one output.

You can picture it as a surface in 3D.

A contour plot is a 2-dimensional drawing that represents a 3-dimensional surface. Instead of showing height with a 3D graph, we show it using curves.

Each curve represents all points (x,y) where the function has the same output value.

Formally, for a function $ z=f(x,y)$, a level curve for height c is the set $ {(x,y):f(x,y)=c}$.

So a contour plot is simply a collection of these curves for several values of c.

- (5 points) Plot the two-variable function:

$$f(x, y) = sin\ (x)cos\ (y)$$

- (5 points) Give the contour plot of this function

[13]:
```
#
# Solution
#
```

[14]:
```
#
# Solution
#
```